SDSM&T
Math Initiative 2016

Mathematics Faculty of the Department of
Mathematics and Computer Science
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Available Online via http://www.mcs.sdsmt.edu/rwjohnso/

Background:
In April 2016 the South Dakota Legislature and Governor Dennis Daugaard approved $250,000 to support the math readiness skills of incoming freshman to the South Dakota School of Mines & Technology (SDSM&T). The effort was spearheaded by SDSM&T President Dr. Heather Wilson in cooperation with members the Department of Mathematics and Computer Science. An earlier summer 2015 pilot program for incoming freshmen – called 'MathSpark,' which included the use of online self-study review materials and faculty mentorship, had shown some success in terms of improved pass rates of these freshman in their College Algebra, Trigonometry and Calculus classes as compared to the previous year's (2014) incoming freshman class (see Appendix A). (*Note:* The report below is limited to efforts on the SDSM&T campus and does not comment, for example, on current outreach to middle schools or high schools.)

Goals:
The overall goal is to increase student success rates in introductory (i.e. 100-level) mathematics courses, specifically:

- Increase the student *pass rates* (meaning C or better) in College Algebra, Trigonometry, Calculus I and Calculus II
- Increase student retention of fundamental skills from one such class to the next

Here are the pass rates for all enrollments starting fall 2010, when the latest admission standards were put into place, through spring 2016[1]:

College Algebra	Trigonometry	Calculus I	Calculus II
66% (674/1015)	**67%** (1532/2288)	**63%** (1829/2917)	**61%** (1532/2495)

Areas Targeted to Achieve these Goals (Brief Overview):

- Improve student study skills ("soft skills") including skills in effective time management, note-taking, active studying and exam preparation
- Improve student core mathematics skills

The key components in the effort to help students in these two areas:

- *Summer Program* for incoming SDSM&T freshmen – Students were provided with online resources and were supported and engaged (virtually) with SDSM&T mathematics faculty prior to fall enrollment. (Student participation was voluntary.)
- *Recitation Sections* for Trigonometry, Calculus I and Calculus II led by SDSM&T mathematics faculty – Students, in classes of limited size, discussed study skills and improved mathematical skills working with SDSM&T mathematics faculty and with their classmates. (A subset of students in the lecture classes for Trigonometry, Calculus I, and Calculus II self-selected into these recitation sections.)
- *Gateway Exams* for Trigonometry, Calculus I and Calculus II – In each class students had two to three gateway exams to pass. These gateway exams focused on fundamental skills

[1] Excludes summer session enrollments.

deemed critical for success in later, not just mathematics, courses. While each gateway could be retaken, perfection or near perfection was required for a student to pass any given gateway. (Gateways were required of all students in Trigonometry, Calculus I and Calculus II.)

Major Findings: The discussion below primarily concerns student pass rate performance in the introductory courses of College Algebra, Trigonometry, Calculus I and Calculus II.

A. Fall 2016 Pass Rates Compared to Earlier Rates

We begin by examining first-time (non-transfer), full-time (12 or more credits) college freshman (i.e. 'cohorts') at SDSM&T. Students entering with AP mathematics credit were excluded. The Fall 2015 cohort, compared to the Fall 2014 cohort, showed near[2] uniform improvement in pass rates for College Algebra, Trigonometry, Calculus I and Calculus II across both semesters and for those repeating a mathematics course spring term after failing to pass this course in the previous fall semester.

Table 1: Pass Rates for Freshman Cohorts 2014-2016

Course	Semester	Pass Rates (Fall Average Math ACT)		
		2014 Freshman Cohort	2015 Freshman Cohort	2016 Freshman Cohort
College Algebra	Fall	65% (ACT 23.4)	⬆ 71% (ACT 22.8)	⬆ 75% (ACT 24.4)
	Spring	42%	⬆ 53%	*
	Spring Repeat[3]	39%	⬆ 53%	*
Trig	Fall	71% (ACT 27.0)	⬆ 77% (ACT 26.5)	⬇ 67% (ACT 26.2)
	Spring	65%	⬆ 71%	*
	Spring Repeat	69%	⬆ 74%	*
Calculus I	Fall	68% (ACT 27.4)	⬆ 81% (ACT 28.3)	⬇ 76% (ACT 28.6)
	Spring	72%	⬆ 75%	*
	Spring Repeat	70%	⬇ 67%	*
Calculus II	Fall	77% (ACT 30.2)	⬆ 79% (ACT 29.7)	⬆ 84% (ACT 28.1)
	Spring	72%	⬆ 75%	*
	Spring Repeat	56%	⬆ 78%	*

[2] See the opening remarks of Appendix A.
[3] A repeat implies a D/F/W in the previous attempt.

For the most recent Fall 2016 cohort, pass rates increased for College Algebra (+4%) and Calculus II (+5%) but decreased for Trigonometry (-10%) and Calculus I (-5%). (The arrows indicate increases/decreases with respect to the previous year.)

Pass rates and, more generally, grades for Trigonometry and possibly Calculus I Fall 2016 may have been affected by a change in the placement process for these courses. See the discussion in the narrative below under "A Few Reflections."

- **Pass Rates and Recitation Enrollment/Performance**

On a space-available basis, all students (not just the 2016 frosh cohort) were allowed to join a Trigonometry, Calculus I or Calculus II recitation section, provided they were enrolled in the corresponding lecture course. While the sample sizes are small, the pass rates for students enrolled in the recitations were uniformly higher than the historic pass rates – especially for those who performed the great majority of the work in the recitation (and were deemed to "pass").

Table 2: Pass Rates – Historic vs. Recitation Students Fall 2016

Course	Historic Pass Rates Fall 2010 – Spring 2016	All Students Enrolled in Associated Recitation	Students who Passed Associated Recitation
Trigonometry	67%	75% (18/24)	88% (14/16)
Calculus I	63%	64% (25/39)	67% (18/27)
Calculus II	61%	78% (25/32)	91% (19/21)

It is illuminating to look at the lecture course grade distributions for the recitation students according to whether they passed or failed the associated recitation. The great percentage of A and B course grades, for example, occurred among those who passed the recitation – 100% (12/12) for Trigonometry, 78% (14/18) for Calculus I, and 79% (15/19) for Calculus II.

Full Grade Distributions for Recitations Fall 2016

Table 3a: Trigonometry Grade Distribution for Trigonometry Recitation Students

Passed Recitation?	Trigonometry Course Grade				
	A	B	C	D	F
No	0	0	4	2	2
Yes	8	4	2	2	0
Totals	8	4	6	4	2

Table 3b: Calculus I Grade Distribution for Calculus I Recitation Students

Passed Recitation?	Calculus I Course Grade				
	A	**B**	**C**	**D**	**F**
No	0	4	3	1	4
Yes	8	6	4	2	7
Totals	8	10	7	3	11

Table 3c: Calculus II Grade Distribution for Calculus II Recitation Students

Passed Recitation?	Calculus II Course Grade				
	A	**B**	**C**	**D**	**F**
No	2	2	2	3	2
Yes	7	8	4	0	2
Totals	9	10	6	3	4

B. Introductory Mathematics Students 2010-2016

The analysis was chosen to begin starting Fall 2010 as this is when more stringent mathematics admission standards were put in place at the South Dakota School of Mines and Technology.

- **Pass Rates**
  The statements below were found to be true regardless of whether the analysis was done with respect to cohorts or all enrollments.
  - Statistically, there was no difference in the pass rates between students with South Dakota as their home state and other students (see Appendix B).
  - There was a significant statistical difference in the year-to-year pass rates of students in College Algebra, Trigonometry and Calculus I. There was generally not a significant statistical difference in the year-to-year pass rates of students in Calculus II. (See Appendix C.)

Table 4: Pass Rates for Freshman Cohorts since Academic Year 2010

Course	Semester	Pass Rates for Freshman Cohorts						
		2010	2011	2012	2013	2014	2015	2016
College Algebra	Fall	73%	80%	67%	62%	65%	71%	75%
	Spring	52%	29%	53%	68%	42%	53%	*
	Spring Repeat	48%	36%	47%	68%	39%	53%	*
Trig	Fall	83%	88%	78%	64%	71%	77%	67%
	Spring	81%	72%	55%	53%	65%	71%	*
	Spring Repeat	86%	45%	50%	40%	69%	74%	*
Calc I	Fall	68%	74%	77%	70%	68%	81%	76%
	Spring	62%	57%	84%	76%	72%	75%	*
	Spring Repeat	69%	55%	81%	77%	70%	67%	*
Calc II	Fall	87%	89%	81%	82%	77%	79%	84%
	Spring	76%	81%	66%	66%	72%	75%	*
	Spring Repeat	67%	80%	67%	47%	56%	78%	*

Table 5: Student Pass Rates, All Enrollments, Since Fall 2010

Course	Semester	Student Pass Rates by Calendar Year						
		2010	2011	2012	2013	2014	2015	2016
College Algebra	Fall	73%	83%	65%	57%	63%	67%	75%
	Spring	*	66%	57%	64%	78%	53%	52%
Trig	Fall	78%	80%	69%	55%	64%	71%	59%
	Spring	*	81%	72%	54%	50%	61%	68%
Calc I	Fall	55%	57%	64%	60%	64%	71%	63%
	Spring	*	54%	53%	78%	72%	67%	64%
Calc II	Fall	56%	60%	57%	63%	60%	62%	60%
	Spring	*	66%	74%	58%	60%	62%	62%

See Appendix E for a graphical display of the results in the above two tables.

C. Precalculus Gateway Score and Course Grade in Calculus I Fall 2016

The mathematics faculty strongly believe that precalculus skills have an important influence on student performance in calculus classes. The first gateway in Calculus I Fall 2016, implemented through the Pearson MyMathTest software, focused on precalculus skills. This gateway could be retaken, up to a deadline, until successfully passed at a 70% level or higher. Failure to pass at at least the 70% level by the deadline resulted in a drop of one letter grade in the course.

The boxplot[4] display below shows the maximal precalculus gateway score obtained by course grade, for all Calculus I enrollments Fall 2016.

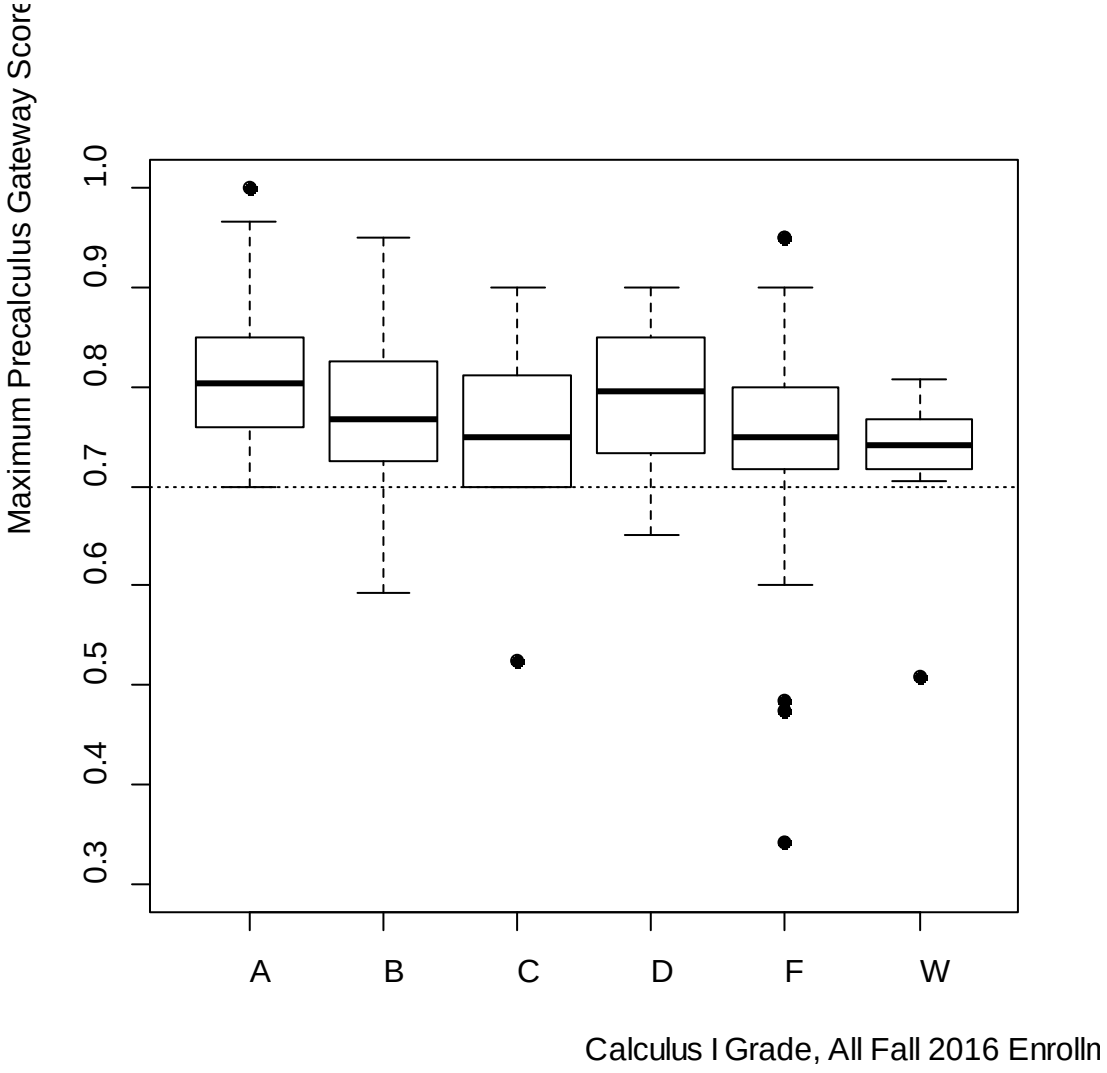

Figure 1: Maximal Precalculus Gateway Score by Calculus I Course Grade

[4] The top of each box displays the 75th percentile, the bottom the 25th percentile, the horizontal line within the box the 50th percentile (or median). Extreme, individual observations ('outliers') appear as dots. The 'whiskers' extend to the most extreme data values that are not outliers.

While the pattern to the side-by-side boxplots does generally show that better grades correspond to higher maximal gateway scores, the gateway scores by students with D grades are a bit of surprise. The median gateway scores for A and D students, in fact, were identical.

The process of repeating the gateway until passed no doubt causes a homogenization of best gateway scores across course grades. Also, only a fairly small number of students (20, or about 7%) earned a D in Calculus I.

Table 6: Maximal Precalculus Gateway Score by Calculus I Course Grade

	Calculus I Course Grade					
	A	B	C	D	F	W
Number	53	64	55	20	65	18
Mean	82	78	76	79	75	73
Minimum	70	59	53	65	34	51
25^{th} Percentile	76	73	70	73	72	72
Median	80	77	75	80	75	74
75^{th} Percentile	85	82	81	85	80	77
Maximum	100	95	90	90	95	81

Reactions:

- **Student Reaction**
  - Summer program students (Summer 2016):
    - About 87% (94 of 108) found the program to be helpful
    - About 95% (103 of 108) would recommend the summer program for future incoming students.
  - Recitation students (Fall 2016):
    - About 60% (44 of 72) believed that the recitation improved their performance in their corresponding class.
    - About 80% (57 of 72) would recommend their recitation class to peers.
    - When surveyed about the most useful and least useful topics in the recitation[5], the MathSpark soft-skills videos[6] were much more commonly included in the "most useful" category rather than the "least useful" category.
    - Recitation discussion on learning styles[7] and 'grit'[8] (i.e. perseverance or determination) were much more commonly included in the "least useful" category than in the "most useful" category.

- **Faculty Reaction**
  - Smaller class sizes in College Algebra and Trigonometry led to less student reticence in class participation and, in particular, asking questions during class.
  - Diagnostic testing often indicated a wide gap between prerequisite material instructors expected students to know and the proficiency students demonstrated in working problems in this material. Fortunately, the diagnostic tests using the Pearson software helps instructors identify areas students have difficulty with. These tests also provide students an early alert regarding holes in their preparation.

[5] Full surveys may be viewed online at http://www.mcs.sdsmt.edu/rwjohnso/ - look for a link about two-thirds of the way down the page.
[6] https://www.youtube.com/playlist?list=PLN2ZOd-6ns4Q5dOe5aC03ZBtLgiVi4fba.
[7] http://www.engr.ncsu.edu/learningstyles/ilsweb.html.
[8] The survey from Duckworth, A.L., Peterson, C., Matthews, M.D., & Kelly, D.R. (2007). Grit: Perseverance and passion for long-term goals. *Journal of Personality and Social Psychology, 9*, 1087-1101 that was used in the recitations may be found at https://www.sas.upenn.edu/~duckwort/images/12-item%20Grit%20Scale.05312011.pdf.

A Few Reflections:

The last few months have been a busy and exciting time for mathematics faculty at SDSM&T making use of the South Dakota Legislature funding to provide additional support to our students in our College Algebra, Trigonometry, Calculus I and Calculus II classes. We are grateful for this support. The curricular discussion among mathematics colleagues fueled by the math initiative support will continue and, in fact, will be reenergized by our investigation of student performance over the last few months.

First, some elaboration on some of the findings mentioned above.

Consider again the students who enrolled in the recitation sections and, in particular, those who did the great majority of the work – in large part focused on the development of 'soft' skills such as study skills, time management, note-taking and exam preparation – thereby earning a 'pass' in the recitation course. An optimistic view of the general success of these students (recall Table 2 and Tables 3a,b,c) is to think of a causal relation being in place; that success in the recitation led to success in the corresponding lecture class. While this may be true in some cases (we certainly hope so!) the truth of the matter is that students self-selected into these recitations. So it may have been commonly the case that those who enrolled in the recitations were among the most highly motivated students. Such students, of course, would tend to perform well both in the recitation and in the lecture. So, for now, we can make the suggestive statement that student recitation success is *strongly associated* with student lecture course success.

The variability in pass rates over the last few years in College Algebra, Trigonometry and Calculus I is undoubtedly due to a number of factors. Three that immediately come to mind are changes to the curriculum, year-to-year variability in student readiness and year-to-year variability in instructor assessment standards. The curriculum in College Algebra and Calculus I has been fairly stable over the last several years with, Calculus I in particular, having a department-wide agreed core of topics and minimal standard use of technology (Maple software) dating back to 2006. Noting student weakness in fundamental algebra skills, our Trigonometry courses over the last couple of years have incorporated a greater amount of this material. The SDSM&T admission process seems to bring, at least in terms of ACT Mathematics score, students with roughly the same average readiness year-to-year (see Appendix D). The above suggests, then, the possibility of some year-to-year differences in instructor assessment standards.

A fourth factor affecting student success, at least for Trigonometry and Calculus I during Fall 2016, concerns changes to the placement process in these courses. In the fall semester we adopted the placement process common to the state system. In previous semesters the cut score on the placement exam to enroll in Trigonometry was higher than the rest of the South Dakota BOR system. This change in placement could certainly be a contributing reason, then, to lower pass rates in Trigonometry Fall 2016. Turning to Calculus I, the "Accuplacer" placement tool is now used across the state system rather than the "COMPASS" exam. Also, as a local change to

just SDSM&T, Trigonometry is now a prerequisite rather than a corequisite for Calculus I. It is a bit unclear how these two changes may have affected grades and the pass rate in Calculus I.

An innovation this semester with respect to our Trigonometry, Calculus I and Calculus II courses is our use of a *uniform* implementation of gateways across all sections – and, so, necessarily – across all instructors, of these three courses. A basic core of fundamental topics was agreed upon for each of these three courses. Using in-department LaTeX mathematical typesetting skills we now have the ability to randomly generate gateway exams which include questions from each fundamental topic area. So the types of questions and their difficulty were uniform across all sections of these three courses Fall 2016. Also, moving forward, the mathematics faculty will certainly want to look back at student performance on these gateway exams to see what refinement, if any, is needed as we continue to use these gateways.

Prior to Fall 2016, by the way, gateways had been in use in all Calculus I sections, but were designed by each instructor for her/his own sections. Gateways had been used by some, but not all instructors, in the Trigonometry and Calculus II sections.

We believe that the fruits of the gateway work will appear in later student coursework in the form of better student skills in the fundamentals. The strong penalty in place Fall 2016 – a drop in letter grade for each failure to pass a gateway – was strong motivation for mastering these fundamental skills.

Second, we present some additional comments on student perspectives with respect to their lower-level mathematics courses. (Please forgive a bit of repetition from earlier in this document.) These perspectives are gleaned from a handful of student surveys and supplement the important, but limited information provided by grade alone. By tracking the response to survey questions over time we may obtain a better sense of how students come to demonstrate mastery in mathematics.

With respect to students enrolled in the Fall 2016 recitations, about 80% would recommend the recitation class to their peers and about 60% believed that the recitation improved their performance in their associated mathematics course. Students in these recitations found the most useful topics to focus on time management and study skills and strategies.

An earlier Spring 2016 electronic survey of students in College Algebra, Trigonometry, Calculus I and Calculus II[9] indicated that students very strongly felt that more active learning (and less lecturing) and smaller class sizes have the greatest potential in promoting student success in mathematics[10]. To a lesser extent, supplemental instruction – including the availability of online materials, was also mentioned. Strategies[11] overwhelmingly mentioned by these students as

[9] 138 students, out of 592, fully responded to this survey. About 15% were in Trigonometry, about 30% were in Calculus I, just over 50% were in Calculus II and the remainder in College Algebra. The full survey may be found online at http://www.mcs.sdsmt.edu/rwjohnso/html/mathinitsurveys.html.
[10] See the results of question 13 at the site given in the footnote above.
[11] Question 10 in this spring survey.

‘highly recommended’ for new students planning on joining our campus were attending class, starting assignments early and taking careful notes and regularly reviewing such after each class. To a lesser extent – in the ‘recommended’ category, asking questions in class, making use of instructor office hours, making use of supplemental instruction and instructional materials and forming study groups, were mentioned.

Finally, a few thoughts about motivation and ‘grit.’

As already mentioned, the 2016 grant brought about a number of additional resources for student success. These included increased opportunities for supplemental instruction and tutoring – with both undergraduate supplemental instructors and faculty. A wealth of online materials, including diagnostic and practice materials, were also available for student use. The *Field of Dreams* model for providing resources would suggest that “if you build it, [t]he[y] will come.” Many of our students did indeed make use of many of these additional resources. Unfortunately, many of our students failed to fully engage with these resources, bringing to mind the old adage “you can lead a horse to water, but you can’t make it drink.” Part of our job as teachers, of course, is to motivate our students to learn new material and – for mathematics teachers in particular, to show how such material will be used in their science and engineering disciplines.

A large part of student motivation, however, must be self-motivation and students also need commitment and perseverance (‘grit’) to master new material. This is the reason for having the discussion on grit within the various recitation sections. While most of the recitation students didn’t have a high opinion of the grit discussion in class, it did resonate with some – especially as individual instructors shared their personal stories of when grit was needed to achieve various goals. The gateway tests, by the way, provided a great exercise machine for grit. Here, a clear goal was established in which multiple attempts were allowed. Students needed to persevere and master fundamental skills to pass each such gateway. Resources aside, a key – if not *the* key for student success, was stated by a student completing the Spring 2016 survey:

> *“My success in math has been more related to my commitment to learn than to any supplemental things.”*

More Detail on How Resources were Provided for Student Success:

Summers 2015 and 2016[12]

- Online materials from Pearson[13], tailored to each course by SDSM&T mathematics faculty, were made available to selected incoming freshman students for the purpose of practice and assessment of mathematics skills[14]. The interactive software provided suggested study plans for students based upon their performance.
- A handful of mathematics faculty each mentored groups of 20-30 students by actively emailing and calling them (including via text using SignalVine) as they progressed through the online materials[15]. Communication also occurred through social media (Facebook and Blackboard, for virtual help sessions).
- College Algebra and Trigonometry review sessions – "Digging in2 Math," primarily geared toward local and regional incoming students, were held by mathematics faculty just prior to fall 2016 orientation.
- For the summer 2015 program: As percentages of fall 2015 enrollments, 30% of the College Algebra students, 22% of Trigonometry students, 31% of Calculus I students and 24% of Calculus II students were enrolled in the summer program.
- The subsequent summer 2016 program had a higher percentage of student participation: As percentages of fall 2016 enrollments, 61% of the College Algebra students, 28% of Trigonometry students, 96% of Calculus I students and 38% of Calculus II students were enrolled in the summer program.

Fall 2016

- Two additional mathematics instructors were hired to reduce class sizes to at most 40 in all 100-level classes – one to help with College Algebra and Trigonometry (previous recent class sizes ranged from 60 to 100 students) and one to help with Calculus I.
- Recitation sections were added – two sections each for Trigonometry, Calculus I and Calculus II (typically 15 to 20 students per section)
    - These recitation sections focused on developing both study skills and math skills. In particular, with some slight variation between sections:
        - Students watched, and then discussed in class, MathSpark videos[16] on time management, writing-up homework, note-taking, active studying and exam preparation.
        - Students spent time, with help from their instructor and classmates, using the Maple computer algebra system as a tool to check their mathematical work and explore mathematical concepts.

[12] 243 students were enrolled in the 2015 summer program, this increased to 467 students in the 2016 program.
[13] MyMathTest was freely provided from Pearson Publishing at http://www.pearsonmylabandmastering.com/northamerica/mymathtest/.
[14] Technical logistics of faculty and student interaction with the online system were resolved courtesy of Prof. Michelle Richard-Greer.
[15] The Pearson software includes an "Ask your instructor" option which directed email to faculty members.
[16] Developed by Dr. Travis Kowalski summer 2015, available at https://www.youtube.com/playlist?list=PLN2ZOd-6ns4Q5dOe5aC03ZBtLgiVi4fba.

        - Instructors assessed student learning preferences and attitudes with surveys in learning styles[17] and determination ("grit").
        - Instructors gathered feedback from students as to what they most valued in the recitation in anticipation of future offerings of such.
        - Substantial time – including most of the second-half of the semester, was devoted to students working with faculty and classmates on solving mathematics problems.
- A uniform "gateway" testing structure was set-up across Trigonometry, Calculus I and Calculus II[18]. These gateways were written entirely by SDSM&T mathematics faculty so as to contain essential, basic material needed for a student to continue to be successful in their mathematics course as well as future science and engineering classes for which their mathematics course is prerequisite[19]. Multiple attempts were allowed for each gateway, but with a final deadline for the last attempt. For any gateway not passed a subsequent attempt could not be made by a student until a recertification process was completed. To be recertified students practiced problems, under the direction of their professor or supplementary instructor (see below), in those areas in which they were found to be deficient. A heavy penalty was put in place for not passing any one gateway. In particular, the cost for not passing any one gateway was a drop in one letter grade for the course.
    - Trigonometry gateways:
        - Basic Trigonometry
        - Inverse Trigonometric Functions
    - Calculus I gateways:
        - Pre-Calculus
        - Differentiation
        - Integration
    - Calculus II gateways:
        - Differentiation
        - Integration
- Upper-level undergraduate supplemental instructors (SIs), two each for Trigonometry, Calculus I and Calculus II were hired. These SIs held scheduled office hours both day and the evening, some at the Tech Learning Center, to assist students with learning mathematics.

[17] http://www.engr.ncsu.edu/learningstyles/ilsweb.html.
[18] Two undergraduate students were hired to assist in the proctoring of these gateway exams.
[19] The gateway test questions were randomly generated with the help of the LaTeX mathematical typesetting program.

Appendix A: Initially Reported Pilot Program Results

The information below was generated by the Department of Mathematics and Computer Science and presented by Dr. Heather Wilson during the SDSM&T fall 2016 faculty/staff convocation. The values in the pass rates columns below differ somewhat from the results presented elsewhere in this report, such as those appearing in Table 1 and Table B.1. This is most likely due to Advanced Placement students being included in Table A.1 but being excluded in Tables 1, B.1. Similar positive improved pass rates from the 2014 cohort to the 2015 cohort, however, are still seen in Tables 1, B.1.

Table A.1: Pass Rates Before and After 'MathSpark' Summer 2015

Course	Semester	Pass Rates		Change
		2014 Freshman Cohort	2015 Freshman Cohort	
College Algebra	Fall	72%	⇧ 81%	+ 9%
	Spring	42%	⇧ 50%	+ 8%
	Spring Repeat	45%	⇧ 67%	+22%
Trigonometry	Fall	72%	⇧ 77%	+ 5%
	Spring	65%	⇧ 70%	+ 5%
	Spring Repeat	55%	⇧ 74%	+19%
Calculus I	Fall	69%	⇧ 81%	+12%
	Spring	72%	⇧ 76%	+ 4%
	Spring Repeat	59%	⇧ 75%	+16%
Calculus II	Fall	77%	⇧ 78%	+ 1%
	Spring	71%	⇧ 74%	+ 3%
	Spring Repeat	29%	⇧ 81%	+52%

Average ACT Math score, by cohort and course, in the above:

Course	Fall 2014	Fall 2015
Algebra	26.4	20.2
Trigonometry	26.4	25.6
Calculus I	25.2	27.1
Calculus II	29.4	29.4

Appendix B: South Dakota Students versus Other Students

When examining student *cohorts*, there is essentially no difference in pass-rate performance between South Dakota students and other students.

Table B.1: Pass Rates for Freshman Cohorts
All Cohort Students Compared with South Dakota Cohort Students[20], 2010-2015

Course	Semester	Pass Rates for Freshman Cohorts - South Dakota Subset in Parentheses					
		2010	2011	2012	2013	2014	2015
College Algebra	Fall	73% (73%)	80% (79%)	67% (64%)	62% (61%)	65% (71%)	71% (73%)
	Spring	52% (83%)	29% (22%)	53% (33%)	68% (64%)	42% (45%)	53% (56%)
	Spring Repeat	48% (78%)	36% (33%)	47% (25%)	68% (64%)	39% (45%)	53% (56%)
Trig	Fall	83% (79%)	88% (83%)	78% (81%)	64% (57%)	71% (70%)	77% (83%)
	Spring	81% (80%)	72% (65%)	55% (51%)	53% (56%)	65% (57%)	71% (69%)
	Spring Repeat	86% (91%)	45% (25%)	50% (43%)	40% (41%)	69% (63%)	74% (50%)
Calc I	Fall	68% (59%)	74% (72%)	77% (80%)	70% (73%)	68% (62%)	81% (82%)
	Spring	62% (68%)	57% (50%)	84% (79%)	76% (70%)	72% (65%)	75% (72%)
	Spring Repeat	69% (67%)	55% (53%)	81% (73%)	77% (60%)	70% (65%)	67% (63%)
Calc II	Fall	87% (88%)	89% (94%)	81% (76%)	82% (79%)	77% (73%)	79% (81%)
	Spring	76% (75%)	81% (83%)	66% (66%)	66% (61%)	72% (76%)	75% (79%)
	Spring Repeat	67% (67%)	80% (0%)[21]	67% (60%)	47% (50%)	56% (71%)	78% (71%)

[20] As determined from the XH.FE.HOME.STATE field (also used in subsequent tables to determine home state).
[21] 0 out of 1.

Table B.2: Pass Rates for Freshman Cohorts – Raw Data and Statistical Tests
Not South Dakota vs South Dakota Students, 2010-2015

Course	Semester	Pass Rates for Freshman Cohorts - Home State: Not South Dakota (Top) vs South Dakota (Bottom)					
		2010	**2011**	**2012**	**2013**	**2014**	**2015**
College Algebra	Fall	40/55 41/56 p=0.954	33/40 37/47 *p=0.786*	35/50 30/47 p=0.518	42/67 34/56 p=0.823	27/47 48/68 p=0.146	23/34 30/41 p=0.601
	Spring	3/13 10/12 ***p=0.005***	2/5 2/9 *p=0.578*	7/10 3/9 *p=0.180*	12/17 7/11 *p=1.000*	5/13 5/11 *p=1.000*	4/8 5/9 *p=1.000*
	Spring Repeat	3/12 7/9 ***p=0.030***	2/5 2/6 *p=1.000*	6/9 2/8 *p=0.154*	12/17 7/11 *p=1.000*	4/12 5/11 *p=0.681*	4/8 5/9 *p=1.000*
Trig	Fall	76/89 62/78 p=0.315	72/78 58/70 *p=0.129*	61/80 58/72 p=0.520	69/99 42/74 p=0.079	86/119 74/105 p=0.767	72/99 52/63 p=0.151
	Spring	41/50 41/51 *p=1.000*	30/38 28/43 *p=0.220*	28/48 19/37 p=0.521	33/65 27/48 p=0.564	36/47 39/68 **p=0.033**	29/40 22/32 p=0.728
	Spring Repeat	9/11 10/11 *p=1.000*	3/3 2/8 *p=0.061*	8/15 3/7 *p=0.680*	10/25 7/17 *p=1.000*	16/21 15/24 *p=0.354*	15/19 2/4 *p=0.540*
Calc I	Fall	72/95 51/86 **p=0.018**	64/84 55/76 p=0.580	77/103 63/79 p=0.428	89/131 58/80 p=0.484	105/144 60/97 p=0.070	96/120 50/61 p=0.751
	Spring	23/41 28/41 p=0.255	26/42 17/34 p=0.298	31/35 22/28 *p=0.316*	46/59 16/23 *p=0.569*	41/53 30/46 p=0.181	31/40 21/29 *p=0.779*
	Spring Repeat	13/18 18/27 *p=0.752*	9/16 8/15 *p=1.000*	13/15 8/11 *p=0.619*	30/37 6/10 *p=0.213*	19/26 11/17 *p=0.737*	11/16 5/8 *p=1.000*
Calc II	Fall	16/19 23/26 *p=1.000*	27/32 30/32 *p=0.426*	39/45 31/41 *p=0.268*	53/63 38/48 p=0.501	46/58 29/40 p=0.434	47/61 43/53 p=0.594
	Spring	61/79 43/57 p=0.810	56/70 45/54 *p=0.653*	53/80 48/73 p=0.948	66/94 39/64 p=0.225	82/117 51/67 p=0.379	75/103 46/58 p=0.360
	Spring Repeat	2/3 2/3 *p=1.000*	4/4 0/1 *p=0.199*	4/5 6/10 *p=0.601*	4/9 4/8 *p=1.000*	4/9 5/7 *p=0.358*	9/11 5/7 *p=1.000*

Small p-values – a commonly used threshold is 0.05 – provide evidence that the two population proportions are different.

Table B.2 Technical Notes:

1. The large-sample test of equality of proportions was used with suitable sample sizes (at least 10 successes and 10 failures in each category). For the small sample case a resample test was used. The corresponding p-values in the small sample case are *italicized*.

2. P-values less than 0.05 are in **bold**. (A lower threshold should probably be used because of the multiple comparison problem. Using a cutoff of 0.05 divided by the number of tests, or 0.05/72, on the *individual* tests guarantees an *overall* Type I error of at most 0.05.)

3. Technically, it is invalid to conduct hypothesis tests on populations (they are conducted on samples to make inference back to the populations).

When examining *all student enrollments,* there is essentially no difference in pass-rate performance between South Dakota students and other students.

Table B.3: Student Pass Rates – All Enrollments
All Students Compared with South Dakota Students, 2010-2015

Course	Semester	Student Pass Rates by Calendar Year - South Dakota Subset in Parentheses					
		2010	**2011**	**2012**	**2013**	**2014**	**2015**
College Algebra	Fall	73% (73%)	83% (83%)	65% (64%)	57% (57%)	63% (66%)	67% (65%)
	Spring		66% (83%)	57% (58%)	64% (63%)	78% (78%)	53% (61%)
Trig	Fall	78% (75%)	80% (76%)	69% (69%)	55% (46%)	64% (63%)	71% (73%)
	Spring		81% (83%)	72% (68%)	54% (57%)	50% (50%)	61% (57%)
Calc I	Fall	55% (49%)	57% (54%)	64% (67%)	60% (59%)	64% (57%)	71% (73%)
	Spring		54% (58%)	53% (51%)	78% (79%)	72% (74%)	67% (63%)
Calc II	Fall	56% (53%)	60% (60%)	57% (51%)	63% (61%)	60% (58%)	62% (62%)
	Spring		66% (61%)	74% (76%)	58% (61%)	60% (58%)	62% (64%)

Table B.4: Student Pass Rates – All Enrollments – Raw Data and Statistical Tests
Not South Dakota vs South Dakota Students, 2010-2015

Course	Semester	Student Pass Rates by Calendar Year - Home State: Not South Dakota (Top) vs South Dakota (Bottom)					
		2010	**2011**	**2012**	**2013**	**2014**	**2015**
College Algebra	Fall	47/64 61/83 p=0.994	36/43 59/71 *p=1.000*	40/60 44/69 p=0.730	44/78 41/72 p=0.947	34/59 53/80 p=0.299	27/39 37/57 p=0.659
	Spring		7/18 24/29 ***p=0.004***	6/11 14/24 *p=1.000*	13/20 15/24 *p=1.000*	17/22 21/27 *p=1.000*	6/15 14/23 *p=0.320*
Trig	Fall	88/109 92/123 p=0.279	82/98 77/101 p=0.191	66/97 75/108 p=0.829	80/128 57/123 **p=0.010**	96/148 97/153 p=0.791	85/124 80/109 p=0.417
	Spring		52/66 79/95 p=0.484	39/50 58/85 p=0.223	31/61 42/74 p=0.491	41/82 39/78 p=1.000	48/71 58/102 p=0.154
Calc I	Fall	91/151 74/151 p=0.049	92/154 87/161 p=0.307	98/160 118/175 p=0.238	104/172 82/139 p=0.792	140/202 87/152 **p=0.019**	128/181 95/131 p=0.728
	Spring		36/73 56/97 p=0.276	49/88 51/100 p=0.521	50/65 63/80 p=0.792	64/90 49/66 p=0.665	58/81 59/94 p=0.216
Calc II	Fall	54/90 52/98 p=0.338	55/92 62/103 p=0.953	74/117 52/103 p=0.056	99/150 89/147 p=0.329	94/152 81/139 p=0.535	100/159 81/131 p=0.853
	Spring		91/131 73/119 p=0.177	87/121 84/111 p=0.514	76/140 97/159 p=0.240	88/141 73/127 p=0.410	109/180 85/132 p=0.490

Small p-values – a commonly used threshold is 0.05 – provide evidence that the two population proportions are different.

Table B.4 Technical Notes:

1. The large-sample test of equality of proportions was used with suitable sample sizes (at least 10 successes and 10 failures in each category). For the small sample case a resample test was used. The corresponding p-values in the small sample case are *italicized*.

2. P-values less than 0.05 are in **bold**. (A lower threshold should probably be used because of the multiple comparison problem. Using a cutoff of 0.05 divided by the number of tests, 0.05/44, on the *individual* tests guarantees an *overall* Type I error of at most 0.05.)

3. Technically, it is invalid to conduct hypothesis tests on populations (they are conducted on samples to make inference back to the populations).

Appendix C: Pass Rates across Years

Going back to Fall 2010, there has been a lack of uniformity in fall term pass rates in College Algebra, Trigonometry and Calculus I. Likewise there has been a lack of uniformity in spring term pass rates in Trigonometry and Calculus I (insufficient data for conclusions on College Algebra for spring term). Calculus II pass rates, however, seem uniform over fall terms and – ignoring one outlier (Spring 2012), over spring terms as well.

Table C.1: Testing Equality of Pass Rates across Years[22]

Class	Term	Years	Cohort Analysis	All Enrollments
College Algebra	Fall	2010 - 2016	$p = 0.040$	$p = 2.44\times10^{-5}$
	Spring			
Trigonometry	Fall	2010 - 2016	$p = 7.20\times10^{-7}$	$p = 6.23\times10^{-10}$
	Spring	2011 - 2016		$p = 1.50\times10^{-9}$
Calculus I	Fall	2010 - 2016	$p = 0.014$	$p = 1.43\times10^{-4}$
	Spring	2011 - 2016		$p = 4.57\times10^{-7}$
Calculus II	Fall	2010 - 2016		$p = 0.590$
	Spring	2010 - 2015	$p = 0.276$	
	Spring[23]	2011 - 2016		$p = 1.82\times10^{-3}$

Small p-values – a commonly used threshold is 0.05 – provide evidence that the pass rates vary across years, for the term in question.

Table C.1 Technical Notes:

A generalized likelihood ratio test was used:

$$-2\ln\Lambda = -2\sum_{i=1}^{y}\left[x_i \ln\left(\frac{\hat{p}}{\hat{p}_i}\right) + (n_i - x_i)\left(\frac{1-\hat{p}}{1-\hat{p}_i}\right)\right] \sim \chi^2_{y-2}$$

(y = number of years, $\hat{p}$ = pooled passing rate, $\hat{p}_i$ = passing rate year i)

[22] If the sample samples were small, then asymptotic theory could not be used and a p-value was not computed. Blank cells correspond to this small sample size situation.

[23] The p-value for all Spring Calculus II enrollments increases to 0.277 if the outlier in Spring 2012 (a high pass rate) is omitted.

Appendix D: Pass Rates and Average ACT Math Score

The following two tables show a fairly stable average mathematics ACT score for any fixed course and term. *Cohorts* are examined in Table D.1, *all enrollments* are examined in Table D.2.

Table D.1: Pass Rates and Average Math ACT for Freshman Cohorts, Academic Years 2010-2016

Course	Semester	Pass Rates and Average Math ACT for Freshman Cohorts						
		2010	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
College Algebra	Fall	73% (22.4)	80% (23.4)	67% (23.0)	62% (23.5)	65% (23.4)	71% (22.8)	75% (24.4)
	Spring	52% (22.0)	29% (21.9)	53% (23.9)	68% (22.8)	42% (22.9)	53% (22.2)	*
	Spring Repeat	48% (22.6)	36% (22.9)	47% (23.7)	68% (22.8)	39% (22.8)	53% (22.2)	*
Trig	Fall	83% (27.2)	88% (26.8)	78% (27.5)	64% (26.3)	71% (27.0)	77% (26.5)	67% (26.2)
	Spring	81% (23.8)	72% (23.7)	55% (23.9)	53% (25.0)	65% (24.7)	71% (23.6)	*
	Spring Repeat	86% (26.7)	45% (26.6)	50% (25.9)	40% (25.8)	69% (25.7)	74% (24.7)	*
Calc I	Fall	68% (28.6)	74% (27.2)	77% (27.4)	70% (26.9)	68% (27.4)	81% (28.3)	76% (28.6)
	Spring	62% (27.0)	57% (26.5)	84% (26.1)	76% (26.4)	72% (26.7)	75% (24.5)	*
	Spring Repeat	69% (27.0)	55% (26.5)	81% (25.8)	77% (26.1)	70% (26.5)	67% (25.7)	*
Calc II	Fall	87% (30.2)	89% (30.3)	81% (30.0)	82% (29.0)	77% (30.2)	79% (29.7)	84% (28.1)
	Spring	76% (28.7)	81% (27.2)	66% (27.7)	66% (27.7)	72% (27.5)	75% (28.2)	*
	Spring Repeat	67% (29.7)	80% (26.6)	67% (28.5)	47% (27.9)	56% (28.3)	78% (29.7)	*

Table D.2: Pass Rates and Average Math ACT
All Enrollments, Since Fall 2010

Course	Semester	Pass Rates & Average Math ACT by Calendar Year						
		2010	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
College Algebra	Fall	73% (22.6)	83% (23.5)	65% (22.8)	57% (23.4)	63% (23.1)	67% (22.9)	75% (23.3)
	Spring	*	66% (22.2)	57% (22.6)	64% (22.9)	78% (22.8)	53% (22.0)	*
Trig	Fall	78% (24.9)	80% (24.1)	69% (25.6)	55% (23.9)	64% (24.2)	71% (24.7)	59% (23.0)
	Spring	*	81% (24.2)	72% (23.4)	54% (22.6)	50% (24.3)	61% (24.2)	*
Calc I	Fall	55% (23.5)	57% (23.1)	64% (23.1)	60% (23.3)	64% (22.7)	71% (24.6)	63% (23.8)
	Spring	*	54% (23.3)	53% (23.5)	78% (23.6)	72% (23.9)	67% (23.1)	*
Calc II	Fall	56% (24.0)	60% (25.3)	57% (24.5)	63% (24.3)	60% (24.4)	62% (24.7)	60% (23.7)
	Spring	*	66% (24.5)	74% (24.5)	58% (23.5)	60% (23.6)	62% (24.4)	*

Appendix E: Visual Display of Pass Rates by Course

Part I: Cohort Pass Rates

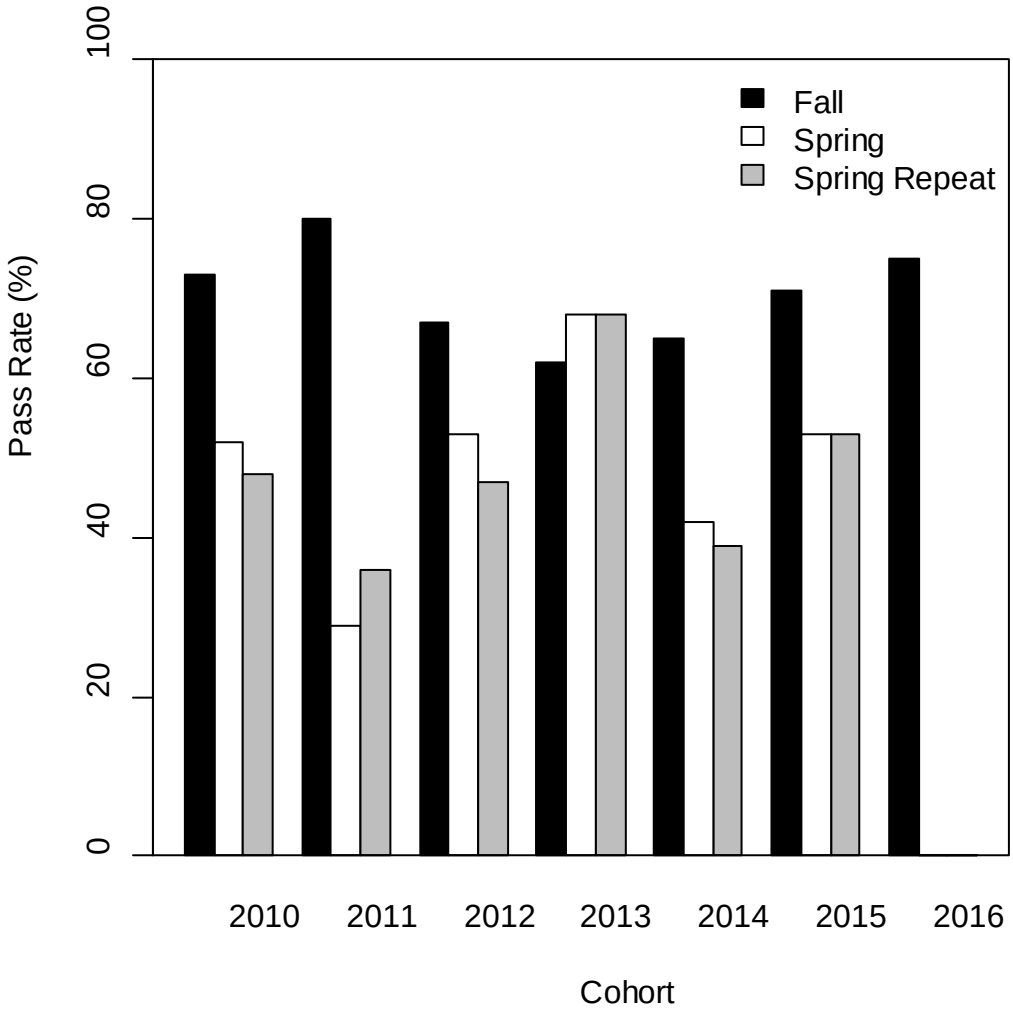

Figure E.1: College Algebra Pass Rate by Frosh Cohort

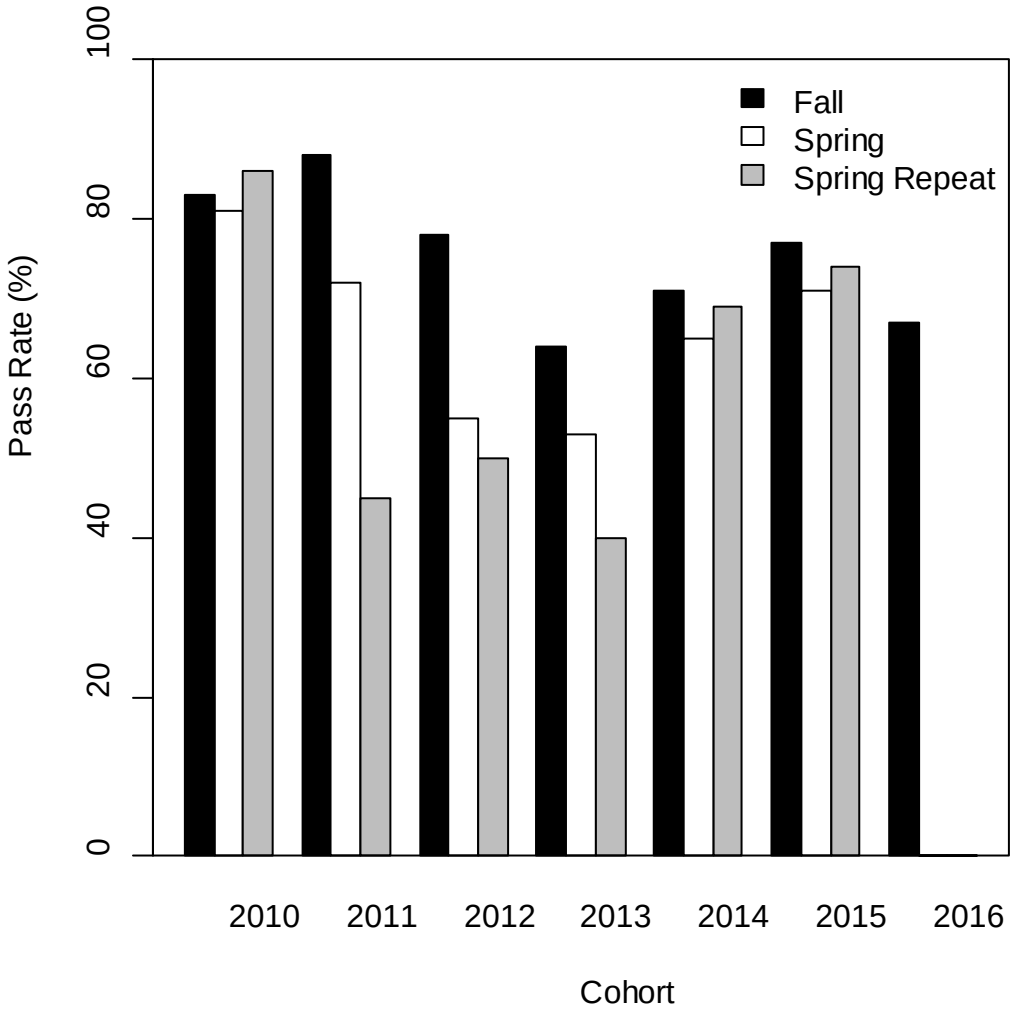

Figure E.2: Trigonometry Pass Rates by Frosh Cohort

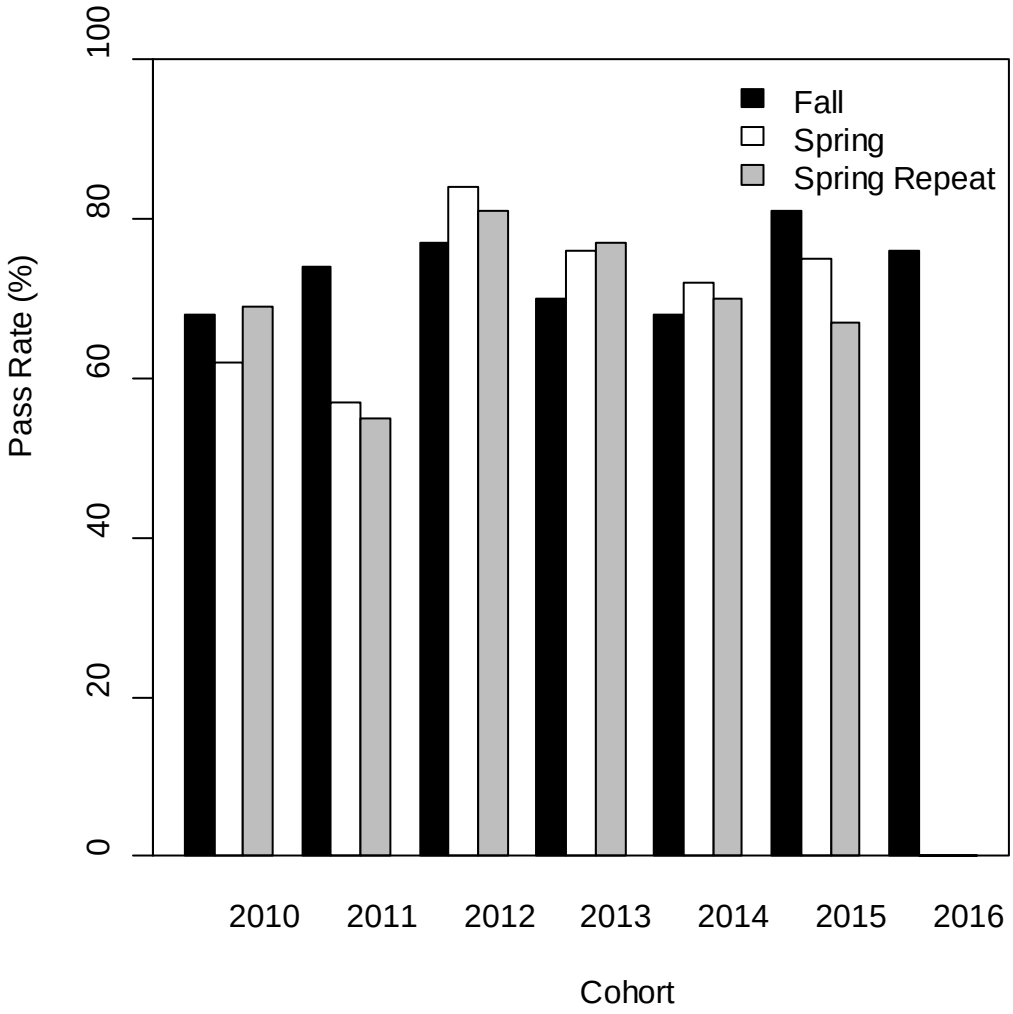

Figure E.3: Calculus I Pass Rates by Frosh Cohort

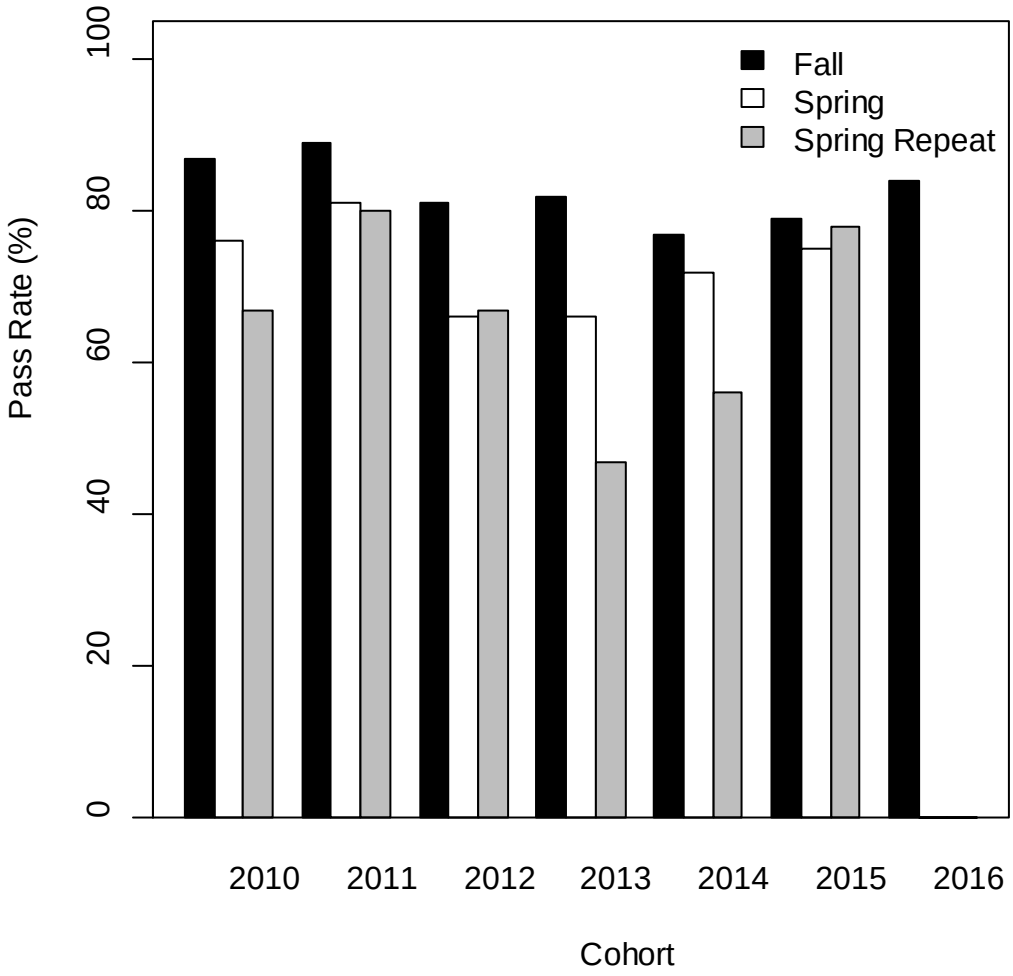

Figure E.4: Calculus II Pass Rates by Frosh Cohort

Part II: All Enrollment Pass Rates

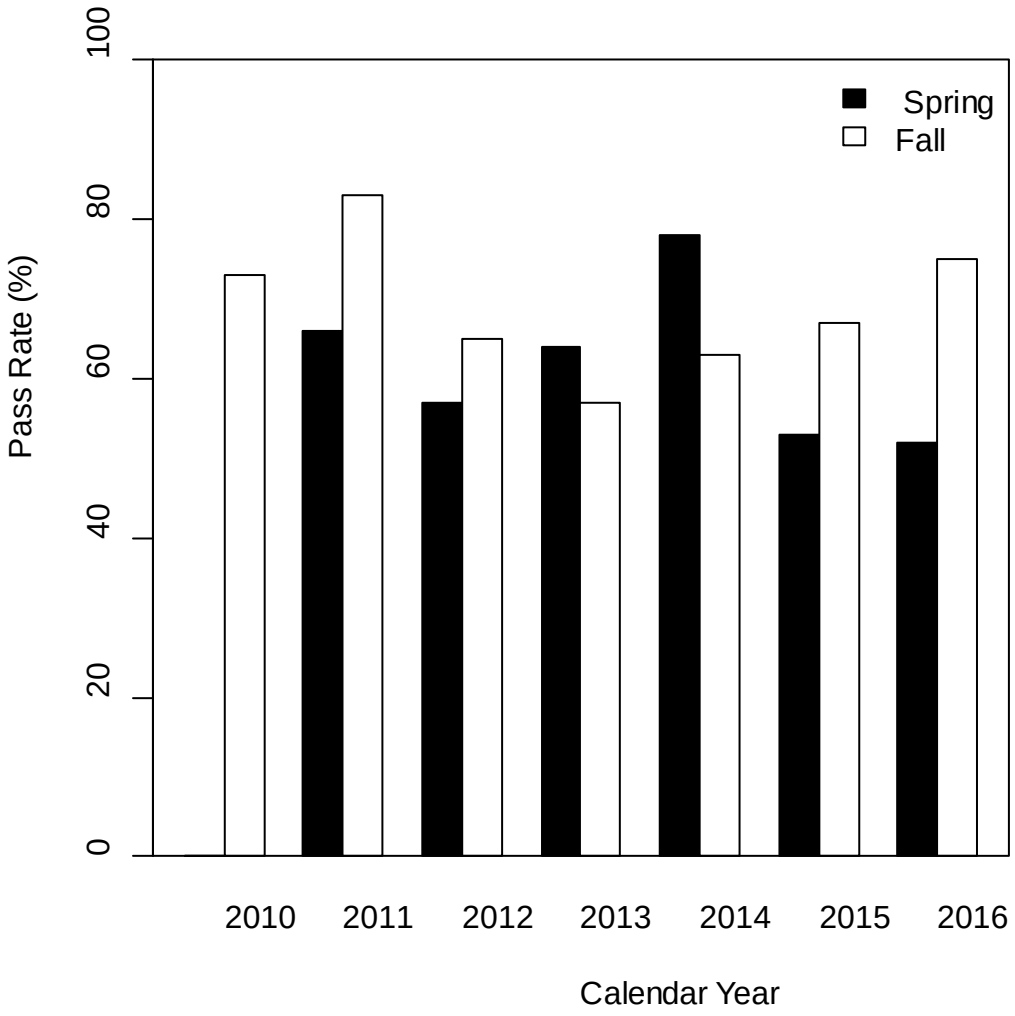

Figure E.5: College Algebra Pass Rates, all Enrollments, by Calendar Year

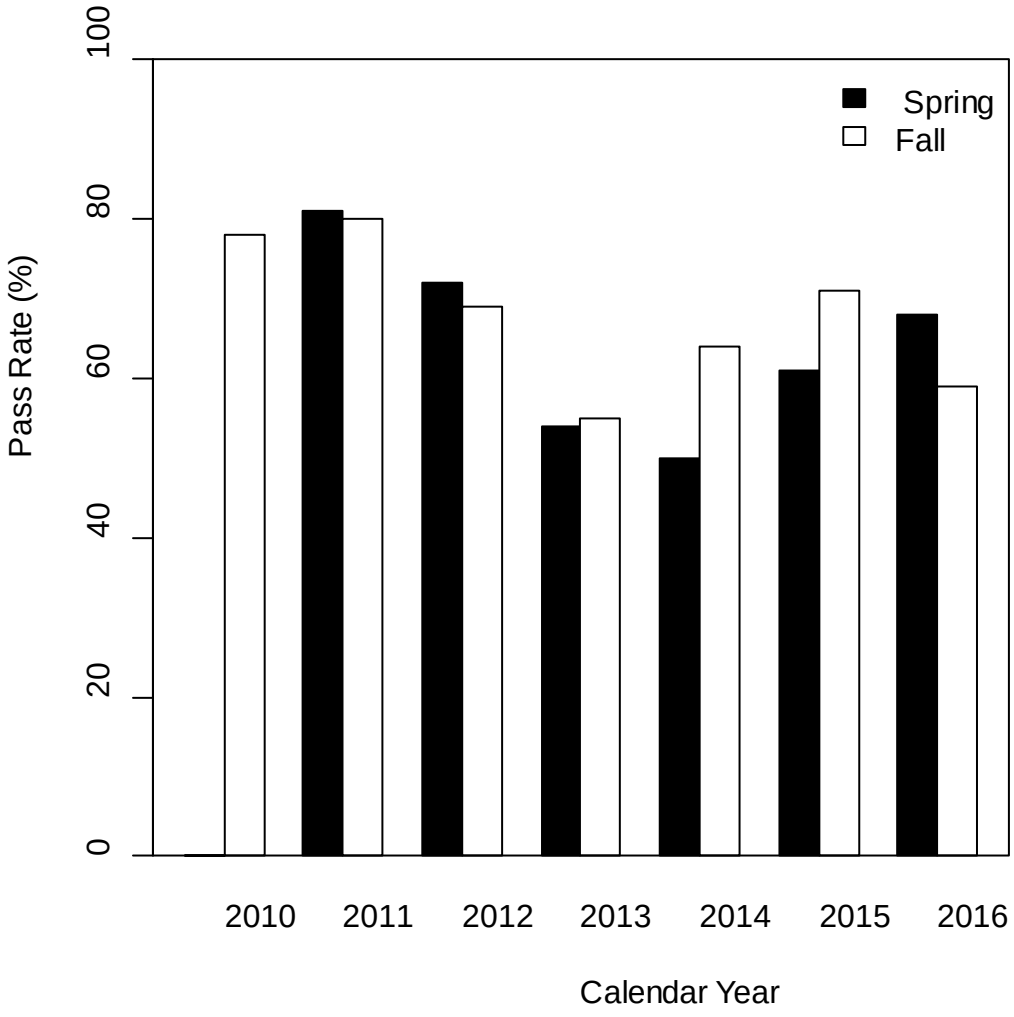

Figure E.6: Trigonometry Pass Rates, all Enrollments, by Calendar Year

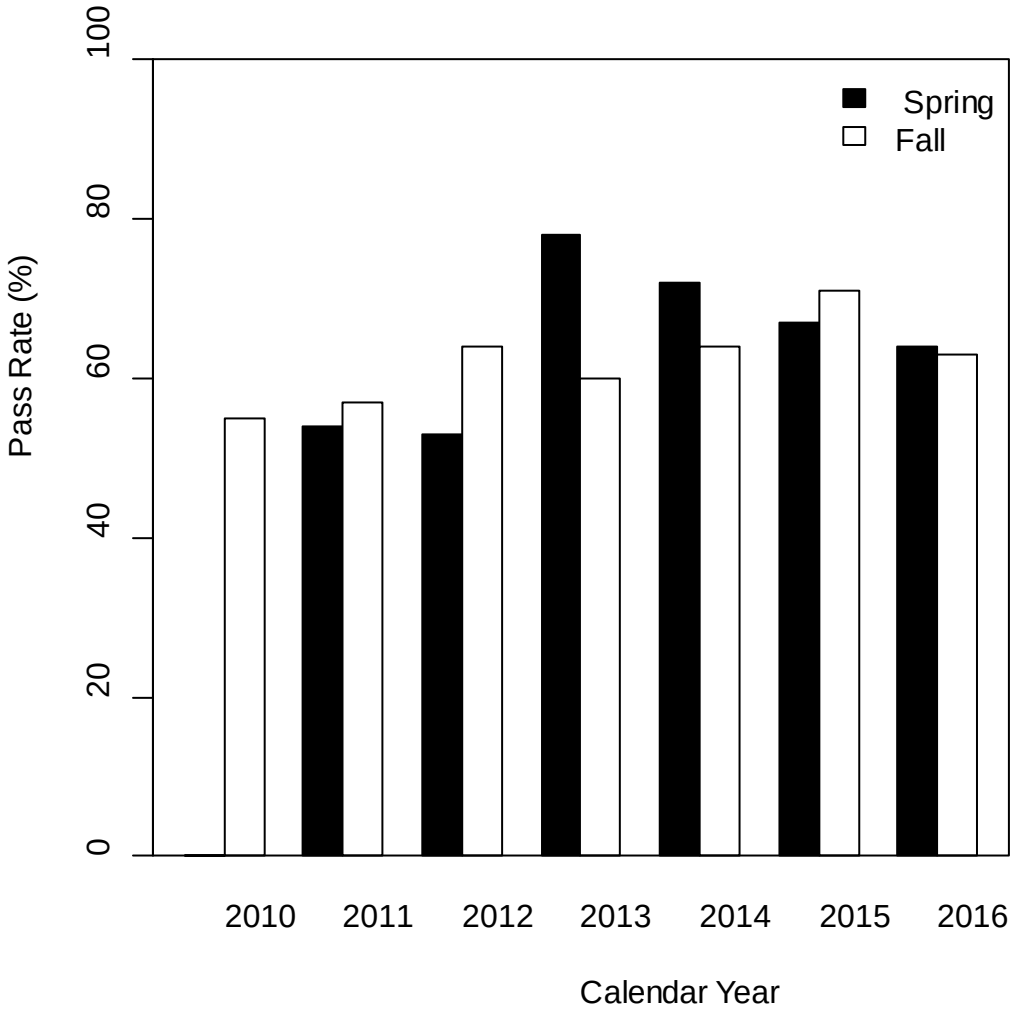

Figure E.7: Calculus I Pass Rates, all Enrollments, by Calendar Year

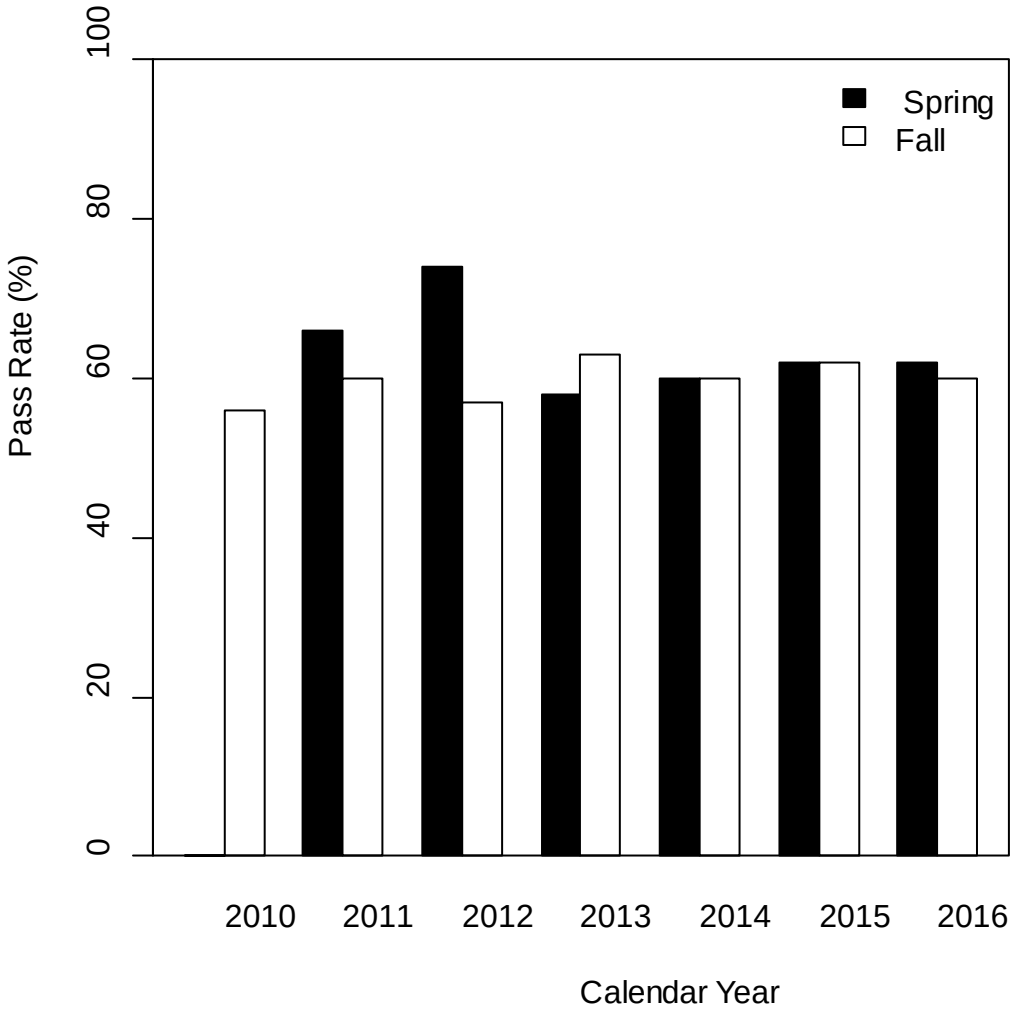

Figure E.8: Calculus II Pass Rates, all Enrollments, by Calendar Year

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